

## 2023 MOCK 2 MATHEMATICS (ELECTIVE) 2

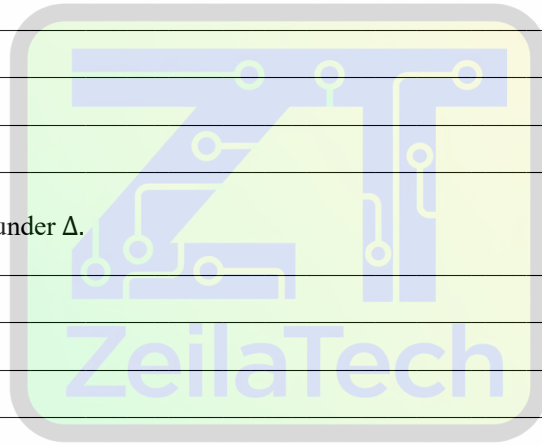
### SECTION A

[48 Marks]

Answer **all** the questions in this section. All questions carry **equal** marks.

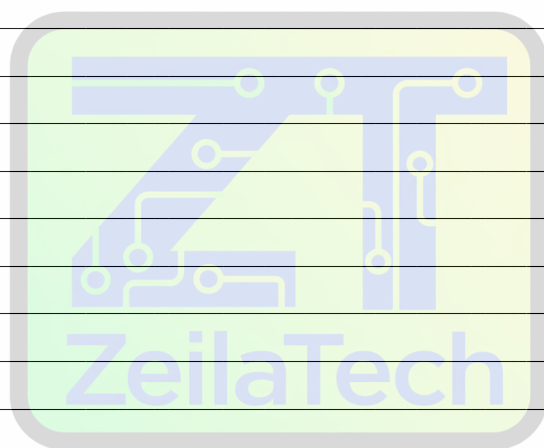
1. A binary operation  $\Delta$  is defined on the set  $R$  of real numbers by  $m\Delta n = m + n + 10$ . Find:
- (a) the identity element.

(b) the inverse of 2 and -5 under  $\Delta$ .



2. Solve the simultaneous equations;
- $$\log(x - 1) + 2 \log y = 2 \log 3,$$
- $$\log x + \log y = \log 6.$$

3. If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $4x^2 + 7x + 3 = 0$ , find the value of  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

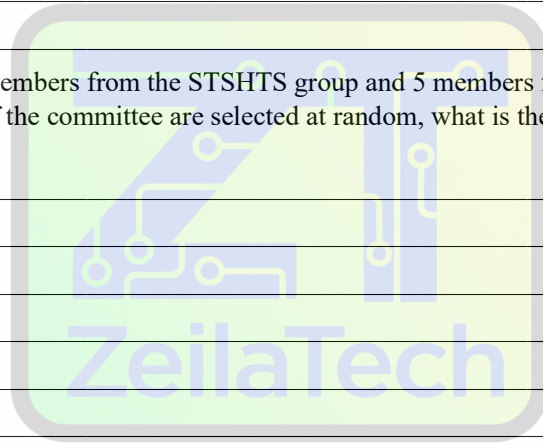


Smart Learning Tools

4. The sum of the first  $n$  terms of a sequence is given by  $S_n = \frac{5}{2}n^2 + \frac{5}{2}n$ , find  
(i) the  $n$ th term of the sequence.

- (ii) write down the first four terms of the sequence.

5. A committee consist of 7 members from the STSHTS group and 5 members from the GRCS group. If a chairman and a secretary of the committee are selected at random, what is the probability that they are of:
- (a) the same group.



Smart Learning Tools

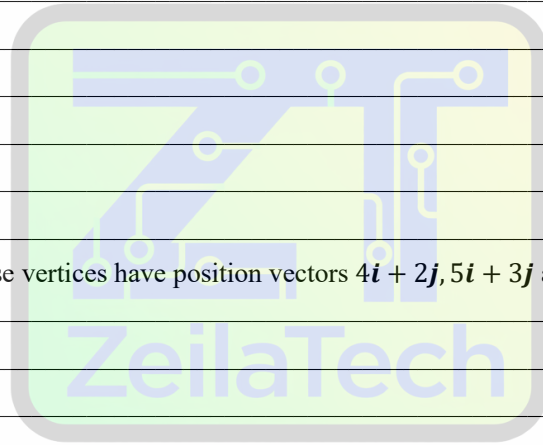
- (b) different groups.

6. The table shows the relation between weekly advertisement (**P**) and the number of items sold (**Q**) by Mr. Affum in his shop.

|   |   |     |    |     |    |    |    |    |    |     |
|---|---|-----|----|-----|----|----|----|----|----|-----|
| P | 0 | 7   | 3  | 5   | 6  | 1  | 8  | 9  | 4  | 2   |
| Q | 0 | 107 | 25 | 101 | 63 | 81 | 42 | 94 | 12 | 105 |

Calculate, correct to **four decimal places**, the Spearman's rank correlation coefficient.

7. Show that the triangle whose vertices have position vectors  $4\mathbf{i} + 2\mathbf{j}$ ,  $5\mathbf{i} + 3\mathbf{j}$  and  $2\mathbf{i} + 5\mathbf{j}$  is isosceles.

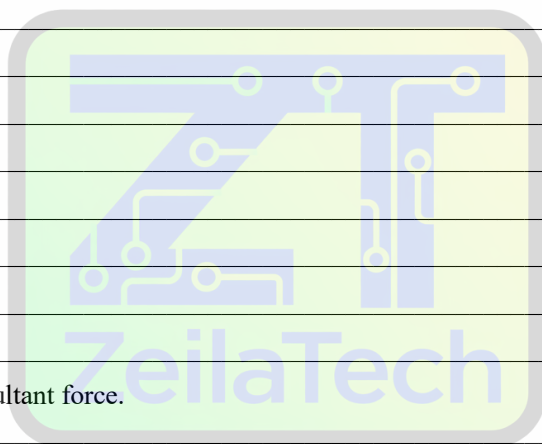


Smart Learning Tools

- (b) Given that  $\mathbf{a} = k\mathbf{i} + 2\mathbf{j}$  and  $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j}$ , find the value of  $k$  for which  $\mathbf{a}$  and  $\mathbf{b}$  are orthogonal

8. Three forces whose magnitude are 15N, 9N and 34N act along the vectors  $-3i + 4j$ ,  $13i$  and  $8i - 15j$  respectively. Find, correct to **one decimal place**,
- (a) the magnitude of the resultant force.

- (b) the direction of the resultant force.



Smart Learning Tools

## SECTION B

[52 marks]

Answer only **four** questions from this section with at **least one** question from each part.

## PART I

## PURE MATHEMATICS

Answer **one** question from this part

9. (a) Express  $\frac{3x^2+7x+28}{x^3+x^2+7x}$  in partial fractions.

(b) If  $7x^2 - 8y^2 + 2xy - 3x = 101$ . Find  $\frac{dy}{dx}$  at the point  $(-2, 5)$ .

10. (a) (i) Write down the matrix **A** of the linear transformation

$$\mathbf{A}:(x, y) \rightarrow (-2x - y, -5x + 3y)$$

(ii) If  $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix}$ , find;

( $\alpha$ )  $\mathbf{B}^2 - \mathbf{A}^2$

( $\beta$ ) matrix  $\mathbf{H}$ , if  $\mathbf{H} = \mathbf{B}^2\mathbf{A}$

(b) Without using mathematical tables or calculator, evaluate

$$\frac{\frac{3}{2} \log 27 - 3 \log 5\sqrt{5}}{\log 0.6}$$

11. (a) A line passes through the centres of the circles  $x^2 + y^2 + 6x + 3y + 4 = 0$  and  $x^2 + y^2 + x - 2y - 3 = 0$ .

(i) Find the equation of the line.

(ii) If the centres of the two circles in (a) are the ends of a diameter of another circle, find the equation of that circle.

(b) If  $\alpha$  and  $\beta$  are the roots of  $3x^2 - 5x - 1 = 0$ , find an equation whose roots are  $\left(\alpha - \frac{1}{\beta}\right)$  and  $\left(\beta - \frac{1}{\alpha}\right)$ .

Smart Learning Tools

## PART II

## STATISTICS AND PROBABILITY

12. The table below shows the distribution of marks of some students in a test conducted by Sir SAMMENS.

| Marks              | 21-30 | 31-40 | 41-50 | 51-60 | 61-70 | 71-80 | 81-90 | 91-100 |
|--------------------|-------|-------|-------|-------|-------|-------|-------|--------|
| Number of students | 2     | 7     | 6     | 7     | 8     | 5     | 3     | 2      |

Using an assumed mean of 65.5,

(a) calculate, correct to **two decimal places**, the mean of the distribution.

(b) calculate, correct to **three decimal places**, the standard deviation.

13. (a) A box contains 7 blue and 4 red marbles. Three marbles are selected at random, one after the other without replacement from the box, find in two decimal places, the probability that;

(i) they are of the same colour.

(ii) two blue and one red marble are selected.

(b) In St. Thomas Senior High Technical School SHS out of every 8 students taught by Sir Solomon and Mad. Esther, attend NSMQ meetings. If 5 students are selected at random from the school, find, correct to **two decimal places**, the probability that:

- (i) **exactly two** attend NSMQ meeting.
- (ii) **less than half** of them attend NSMQ meeting.
- (iii) **at least** three of them attend NSMQ meeting.

### PART III

#### VECTORS AND MECHANICS

14. The position vectors of the points  $A, B, C, D$  relative to the origin,  $O$ , are given by  $\vec{OA} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ ,  $\vec{OB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ ,  $\vec{OC} = \begin{pmatrix} 4 \\ n \end{pmatrix}$  and  $\vec{OD} = \begin{pmatrix} p \\ -1 \end{pmatrix}$ , where  $n$  and  $p$  are constant. Find the:

- (a) unit vector in the direction of  $\vec{AB}$
- (b) values of  $n$  for which angle  $AOC = 90^\circ$
- (c) values of  $p$  for which the length of  $\vec{AD}$  is  $\sqrt{37}$  units.

15. (a) A school bell weighing 75N is suspended by two ropes which make angles  $42^\circ$  and  $67^\circ$  with the rope respectively. Find the tensions in the rope, correct to **two decimal places**.

(b) A uniform plank  $AB$ , 10cm long and mass,  $mg$ , rest on two supports  $A$  and  $B$ . A load of  $8kg$  is placed on the plank at  $C$ , such that  $|AB| = 3.5m$ . The reaction at  $B$  is 100N. If the system remains in equilibrium under the action of these forces, calculate;

- (i) value of  $m$ ; in **one decimal place**.
- (ii) the reaction at  $A$ .

[Take  $g = 10ms^{-2}$ ]

ZeilaTech  
Smart Learning Tools

**END OF PAPER**